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ABOUT SOLVING ABILITY OF THE SELF-CONSISTENT METHOD RELATIVE TO THE VLF INVERSE PROBLEM AT NONSTATIONARY CONDITIONS

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Abstract

The results of numerical analysis of an abnormal electric conductivity appearance in the middle polar atmosphere caused with an ultrarelativistic electron precipitation (~100 MeV) are represented. A sporadic D-layer of conductivity in the middle auroral atmosphere is caused with this precipitation. The investigation is fulfilled due to the PGI KSC RAS experimental data for the completely auroral Aldra - Apatity radio pass with the 885 km length and three frequencies in 10 – 14 kHz band. An evaluation of effective height *h* dispersion was obtained.

1. Introduction

The investigation of relativistic and ultrarelativistic electron precipitations from magnetosphere and cosmos into the Earth atmosphere began nearly 40 years ago with the indirect methods (the measurements of the deceleration X-ray radiation of electrons with the help of the aerostats [1] and the on ground VLF measurements [2, 3]) and by the direct methods (registration of the electron fluxes on the rockets in the atmosphere [4] and on the cosmos ships [5]). However, the continuous monitoring of the ultrarelativistic electron fluxes (with energy near 100 MeV) in polar cosmos has not been carried out until now, and thus the on ground VLF method is still the singular one to register this type of electrons. Unfortunately, the VLF beacons in the polar region have finished their regular operating activity nearly 1990s.

Analyzable in present research experimental data of the RAS PGI present the relative variations of amplitudes E_i and the phases φ_i of the radio signals, that were received in Apatity from beacon in Aldra (Northern Norway) at three frequencies of VLF range: f_i = 10,2; 12,1 and 13,6 kHz (*i* = 1, 2, 3), Fig. 1. The length of the radio pass is 885 km and it is completely auroral. The region of ionosphere essential for radio reflection is a regular D-layer of conductivity. But in a case of ultrarelativistic electron precipitation a reflecting layer appears at abnormally low altitudes (from 10 to 40 km). It is an irregular sporadic D-layer of conductivity [2, 3].

The conductivity disturbance of the middle polar atmosphere on 30 of April 1992 (13:00 – 13:30 UT) caused with these electrons is analyzed in the current research. An "Earth – ionized atmosphere layer" wave guide is considered as isotropic one during all moments of a disturbance, since radio signal propagation took place at the daytime conditions. The numerical analysis was performed on the basis of VLF inverse problem solved by the self-consistent method [6].

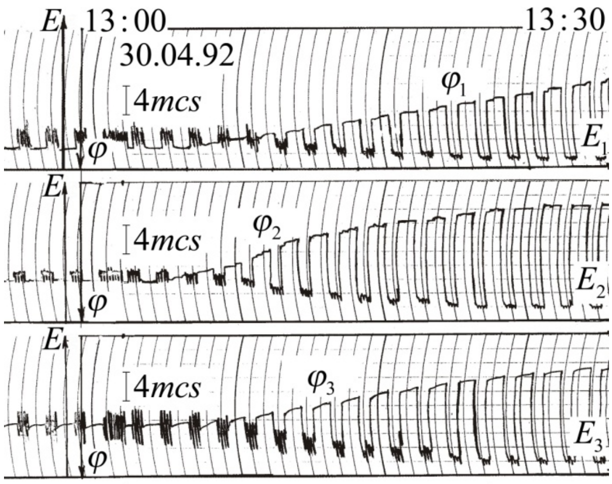

Fig. 1. Experimental data. The relative variations of amplitudes E_i and phases φ_i of the radio signals at three frequencies (i = 1, 2, 3) during the abnormal disturbance of conductivity on 30 April 1992.

2. The self-consistent method used to a VLF inverse problem

A theory of very low-frequency wave propagation in a wave guide makes it possible to establish an implicit analytical relation between the measured amplitudes E_i, phases φ_i of the radio signals and several required parameters characterizing the electric properties of an upper reflecting layer. In previous investigations [2, 3] the reflection coefficient module of the first ionosphere ray *R(t)* and the effective height *h(t)* were used as the unknown parameters. The effective altitude definition relates to a radio signal frequency rigidly. The effective altitude depends on the frequency weakly. This dependence may be estimated by a numerical analysis if a conductivity profile of atmosphere and low ionosphere as function of the altitude is known. But such profile is not available in the cases of the polar abnormal disturbances. So the purpose of our investigation is a searching of an answer on the following question. Are the quality of the experimental data and the solving accuracy of the inverse problem high enough to evaluate the frequency dispersion of the effective height *h(t)*?

The relation between the parameters sought for and the quantities measured can be considered as linear, assuming smallness of the parameter variations (ΔR_n и Δh_n) and the time intervals Δt_n at which the disturbance time is divided (*n* = 1, 2,…, *m*). Thus the system of linear equations was solved for each temporal step:

$$\begin{cases} \dfrac{\Delta E_{in}}{E_{in}} = \dfrac{\Delta R_n}{W_{in}} \dfrac{\partial W_{in}}{\partial R} + \dfrac{\Delta h_n}{W_{in}} \dfrac{\partial W_{in}}{\partial h}, \\ \Delta\varphi_{kn} = \Delta R_n \dfrac{\partial \Phi_{kn}}{\partial R} + \Delta h_n \dfrac{\partial \Phi_{kn}}{\partial h} \end{cases}$$

where the experimental relative amplitude variations $\dfrac{\Delta E_{in}}{E_{in}}$ and the phase variations $\Delta\varphi_{kn}$ are in the left part. The numerically calculated partial derivations of the attenuation function modulus *W* and of the attenuation function argument Φ_{kn} with the fixed values of their arguments $R(t_n)$, $h(t_n)$ are in the right part.

The complex attenuation function *W* is a ratio of the electric field strength of a point source in a point of observation, placed into an actual waveguide (Earth – ionosphere in our case), to the electric field strength of the same source in a reference model. It can be free space [7] or the space above a perfectly conducting surface [8]. We use the last definition. The attenuation function was represented as the superposition of the diffraction wave on ground [9] and two ionosphere rays (once and twice reflected):

$$W(D,f) = W_0(D,f) +$$
$$+\frac{1}{2}\alpha_1\left(1+R_g(I_1)\right)^2 \sin^2(I_1)\left|R(\psi_1)\right| e^{i[\kappa_1 + \arg R(\psi_1)]} +$$
$$+\frac{1}{2}\alpha_2\left(1+R_g(I_2)\right)^2 R_g(I_2)\sin^2(I_2)\left|R(\psi_2)\right|^2 e^{i[\kappa_2 + 2\arg R(\psi_2)]},$$

where $W_0(D,f) = 0.65e^{i1.0}$ with accuracy of 10% in the present case; *D* is a radio pass length; *f* is the frequency of received signal; R_g is a Fresnel reflection coefficient from the earth – air boundary; *R* is a Fresnel reflection coefficient from the top boundary of the model waveguide with a height *h* as a function of frequency and an angle of wave incidence is ψ_q (*q* = 1, 2) counted from the ionosphere normal; κ_q is a dimensionless difference between the optical pass of a ray with number *q* and the optical length of an on ground source–receiver arc. This difference is a function of the effective height *h* and the Earth radius; α_q is a coefficient taking into account the focusing and defocusing action of the spherical reflecting surfaces [7, 8]; I_q is an angle of ray source exit, counted from the normal to earth boundary. Time dependence is $e^{-i\omega t}$.

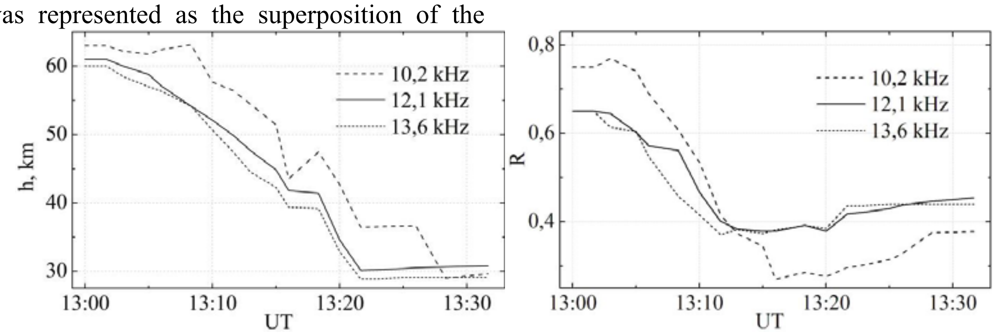

Fig. 2. Frequency dispersion (in the radio band 10 – 14 kHz) of the effective height h and the reflection coefficient modulus R, depending on time, at a case of ultrarelativistic electron precipitation into polar atmosphere on 30 April 1992.

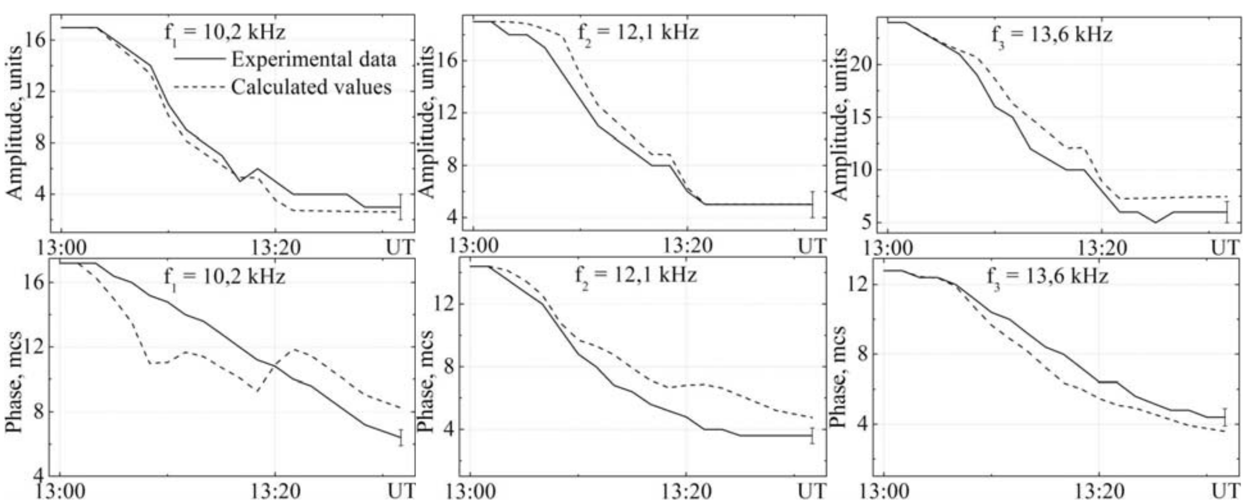

Fig. 3. Comparison of the measured and calculated values of the signal amplitudes and phases at three frequencies of the radio band 10 – 14 kHz as a result of the VLF inverse problem number 2 solving. The solid curves are the experimental data.

The values of required parameters can be easily obtained at any moment of the disturbance due to their continuity, if we know their initial values h_0 and R_0 at some moment of time. The according system of equations is following:

$$h_m = h(t_m) = h_0 + \sum_{n=1}^{m} \Delta h_n ,$$

$$R_m = R(t_m) = R_0 + \sum_{n=1}^{m} \Delta R_n .$$

The summation here is according to the time intervals at which the time interval of distucrbance is divided. In compliance with the linear approximation a time step for summation should be so small (100 s in present case) that the experimental relative amplitude variations and phase variations are small enough at each time step for the correctness of linear approximation using. It is possible to choose any moment of disturbance *t'* as the initial moment for the analysis, which can be fulfilled at any direction of time. In present analysis $t' = t_0$, where t_0 is a moment previous to the disturbance. But the values $R_0 = R(t')$ and $h_0 = h(t')$ are unknown quantities. It is sufficiently to minimize relative to these parameters the functional *G*, which represents the discrepancy between measured and calculated signal variations in compliance with the existing wave propagation theory. In the present case the discrepancy functional is of the form:

$$G = \sum_i \sum_{n=1}^{m} \left[\frac{\left(\tilde{W}_i(t_n) - E_i(t_n)\right)^2}{E_i^2(t_0)} + \frac{\left(\tilde{\Phi}_i(t_n) - \varphi_i(t_n)\right)^2}{\left(\Delta\varphi_i\right)^2} \right],$$

where *i* is a frequency number; *m* is a number of time intervals, at which the disturbance is divided; $E_i(t_n)$ is the measured signal amplitudes expressed in conventional units; $\tilde{W}_i(t_n)$ are the calculated and normalized absolute values of the attenuation function modulus $\tilde{W}_i(t_n) = \frac{W_i(t_n)E_i(t')}{W_i(t')}$; $\varphi_i(t_n)$ are the measured signal phases; $\tilde{\Phi}_i(t_n)$ are the calculated and normalized values of the attenuation function argument $\tilde{\Phi}_i(t_n) = \Phi_i(t_n) - \Phi_i(t') + \varphi_i(t')$; $\Delta\varphi_i$ is the maximum signal phase variation during the time interval of disturbance. It is necessary that the number of measured quantities in the functional *G* should be more than the number of required parameters. Thus the algorithm autonomy from any a priori idea of the ionizing agent nature is achieved.

Inverse problem may be solved in the negative time direction in order to evaluate the error of method by comparison with the results of analysis in the positive time direction. In such case the time moment corresponding to the disturbance maximum may be chosen as the initial moment of time t'. The preference is given to the solution, which provides an essentially less value for the discrepancy functional. The direct time solutions had such property for all three variants of inverse problem, which we shall describe lower. The functional *G* values differed more than 8 times for *the solutions* of the inverse problem in opposite time directions.

3. Three versions of the VLF inverse problem

Three versions of the VLF inverse problem were solved in order to appreciate the effective height frequency dispersion due to the experimental data. These versions differ from each other by their effective height definition. The effective height of the waveguide channel "earth – ionized atmosphere layer" we call an altitude, relative to which the complex reflection coefficient is a real and negative quantity at a fixed frequency and for the sliding angles of wave incidence. That is:

$$\arg R(\psi, f_i) = \pi .$$

The approximate formulas, postulated as the result of numerical calculations, were used to calculate the complex reflection coefficient argument for the neighboring frequencies:

$$\arg R(\psi, f_k) = \pi \pm \delta \frac{\frac{\pi}{2} - \psi}{\frac{\pi}{2} - 1},$$

where ψ is an angle of wave incidence on the upper reflective layer counted from the normal to it ($\psi > 1$); $\delta = 0.35$ rad.; *i* ,*k* = 1, 2, 3. Sign "+" corresponds to the case of $f_i < f_k$ and sign "-" corresponds to the case of $f_i > f_k$. The dependence of complex reflection coefficient modulus on the frequency was neglected but the dependence on ψ was linear with one free parameter [6]. The effective height was defined relative to the first operating frequency 10.2 kHz in the first inverse problem, and the discrepancy functional contained two amplitudes and two phases for the first operating frequencies 10.2 and 12.1 kHz (*i* = 1, 2). The effective height was defined relative to the third operating frequency 13,6 kHz in the third inverse problem, and the discrepancy functional contained the amplitudes and the phases for the last two operating frequencies 12.1 and 13.6 kHz (*i* = 2, 3). The effective height was defined relative to the middle operating frequency 12.1 kHz in the second inverse problem, and functional *G* contained the amplitudes and the phases for all three operating frequencies (*i* = 1, 2, 3).

Fig. 2 shows the effective heights and the modulus of first ray reflection coefficients found from three above described inverse problems as the functions of time of disturbance caused by the ultrarelativistic electrons precipitation into the middle atmosphere. As we can see the disturbance analyzed is characterized by the anomalously small values of effective height, which collapsed to 30 km. For comparison we notice that it lowered to the values of 45 – 50 km during the most energetic sun proton precipitations [2]. The present disturbance has also anomalous values of the reflection coefficient module of the first ionosphere ray, which decreased in 1,5 - 2 times. According to the Maxwell equation solution by a computer none of the monotonous electric conductivity layers can give so small values of the reflection coefficient for the sliding angles of incidence, which is obtained in present case. Thus the real layer is not monotonous, that signifies an appearance of a sporadic D-layer of conductivity in the middle atmosphere. According to such smallness of the reflection coefficient the contribution of the ray, twice reflected from the upper conductivity layer, is a negligible quantity in a receiving signal.

As one can see from the left part of Fig. 2, three curves do not have any random chaotic crossings and that the effective height values as the functions of frequency form one and the same sequence at each moment of time: $h(f_1) \geq h(f_2) \geq h(f_3)$. It means that the quality of experimental data has allowed us to estimate the effective height frequency dispersion in the radio band 10 – 14 kHz.

There is a usability condition for the *W* formula [7], used by us, which is expressed by the inequality:

$$\cos(I_q)kh >> 1 .$$

We must notice that fulfillment of this inequality is getting worse in a case of smaller values of *h*. Therefore the results for small values of the effective height have an estimating character and have no sense near 25 km. The usage of the geometry optic description for the first ionosphere ray is forbidden.

4. Conclusions

The affirmative answer for the formulated question about the effective altitude frequency dispersion in the radio band 10 – 14 kHz was obtained as the result of this research. This result gives an opportunity to increase the accuracy of the VLF inverse problem solving by introducing at this problem a third parameter sought for, characterizing the frequency dispersion of an effective height or of the reflection coefficient modules.

Fig. 3 represents the comparison of experimental and calculated values of the radio signal amplitudes and the phases at all three frequencies applicable to the second inverse problem solving. The experimental and calculated curves have the mutual beginning due to the normalization pointed above. The experimental data error is equal to ± 1 of the conventional units for the amplitudes and $\pm 0,5$ mcs for the phases. Thus the obtained result allows us to hope to improve the represented nearness between the experiment and the calculations for not stationary conditions of the ultrarelativistic electron precipitations.

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